Two-Dimensional Array Processing with Compressed Sensing

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***Abstract*— Front-view radar images of large moving targets such as humans are difficult to capture with existing radar techniques such as SAR and two-dimensional array processing. SAR images may be significantly distorted by the motion of the human. Array processing may require a large number of sensors which may make the radar costly and complex. In this paper, we propose to reduce the number of sensors required for generating frontal images by combining compressed sensing with array processing. First, we apply a compressed sensing based reconstruction technique to generate frontal images of a moving phantom target on a frame to frame basis. In the second technique, we exploit the temporal correlation across multiple frames using Kronecker compressed sensing to generate the radar image of each frame. Both approaches are implemented with 25% and 50% of the number of sensors that would be required by conventional two-dimensional array processing.**

I. Introduction

High resolution top-view radar images of ground based and aerial targets have been generated for several decades. The radars have used wideband pulses for gathering downrange information and one dimensional apertures for gathering azimuth information [1]. However, frontal images convey more information than top view images for some radar targets such as humans for applications such as biometry and security and surveillance operations. Conventionally, front view images of radar targets have been generated using synthetic aperture data [1]. However since humans are rarely still, significant distortions can be introduced in the SAR data. Instead, in 2006, Lin implemented a three-element continuous wave Doppler radar to generate a frontal image of a moving human [2]. He first resolved the different body parts based on their micro-Dopplers. Then he estimated the azimuth and elevation of each body part using a three-element receiver. This low-complexity solution is successful in generating the frontal image of a human provided the micro-Dopplers are well resolved. When that is not the case, the images are blurred and distorted. Alternately, the radar images can be generated with two-dimensional array processing with large antenna apertures. For instance, in order to resolve a large target of the size of $1m \times 1m$, at a standoff distance of 1m, we would require a $10\lambda \times 10\lambda$ aperture to get a resolution of 5° where λ is the wavelength of the radar. If the elements in the aperture are spaced at 0.5λ, then the number of elements required is 400. Therefore, two-dimensional array processing of large targets involves a large number of sensors and is hence both costly and complex. In this paper, we investigate the possibility of reducing the number of sensors required for imaging large moving objects by incorporating compressed sensing (CS) principles into two-dimensional array processing.

Recent investigations in CS in radar can be broadly classified into two categories. In the first category, belongs the studies which are theoretical in nature. And the second category constitutes of studies of applied nature. For instance, in some of the earliest studies in CS in radar, [3] and [4], the problem was to detect 'k' moving targets by a radar with $N \times N$ number of sensors. The contribution of this work is to propose an upper-bound on the number of moving targets the radar can detect based on certain stylized pulses. In [5], the authors propose a greedy algorithm for certain radar reconstruction problems which the authors claim, outperform standard l_1-minimization techniques. There are some studies, like [6], [7], [8], which use concepts of sparsity and incoherence in radar applications. However, these are not truly CS works. This is because, these papers propose adaptive update techniques for the measurement operator. This goes against the very philosophy of CS which hinges on the belief that the measurement is data independent and non-adaptive. Most of the applied studies address the problem of synthetic aperture radar (SAR) imaging [8], [9], [10], [11], [12]. In this scenario, the problem is to reconstruct an image from its tomographic projections. CS for tomographic reconstruction is a well studied problem in other branches, e.g. medical imaging, microscopic imaging. Radar is a different application area, but the underlying mathematics for reconstruction remains the same.

We propose an alternate technique to CS based SAR to image moving radar targets. Through the use of CS, we reconstruct a radar image with reduced number of sensors for the same antenna aperture size as array processing. In Section II, we briefly introduce the principles of CS. In Section III, we simulate the frontal image of a phantom moving radar target using two-dimensional array processing. We investigate the possibility of reconstructing this radar image with reduced sensors using two CS techniques. The first technique reconstructs the radar image at each time

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instant, or frame, piecemeal. The second technique exploits the inter-frame temporal correlation using Kronecker compressed sensing to reconstruct each frame. Both techniques are tested for 50% and 75% reduction of the total number of sensors and the reconstruction errors are presented.

II. Brief Review of Compressed Sensing

Compressed sensing (CS) studies the subject of solving an under-determined system of linear equations where the solution is known to be sparse. Formally, this is expressed as follows,

$$y_{m\times 1} = A_{m\times n}x_{n\times 1}, m < n \tag{1}$$

where x is the signal, A is the measurement basis and y is the measured data. The inverse problem (1), in general has infinitely many solutions; but what if the solution is known to be sparse? In his seminal paper [13], Donoho showed that for most large under-determined systems, a sparse solution is also unique. This implies that in order to find the unique sparse solution, one may as well seek the sparsest solution. Thus, it is natural to solve (1) by the following l_0-norm minimization problem,

$$\min_x \|x\|_0 \; subject\, to\, y = Ax \tag{2}$$

where l_0-norm counts the number of non-zeroes. Unfortunately l_0-norm minimization is an NP hard problem and hence is not applicable in practical situations [14]. CS proves that it is possible to substitute the NP hard l_0-norm by its closest convex envelope, the l_1-norm, and still be able to recover the correct sparse solution [13], [15], [16]. Thus, instead of employing (2) to solve (1), one can employ l_1-norm minimization,

$$\min_x \|x\|_1 \; subject\, to\, y = Ax \tag{3}$$

where the l_1-norm is the sum of absolute values. This can be easily solved by linear programming with a complexity of $O(n^3)$. Also, recent research efforts have resulted in the development of many algorithms that can solve the l_1-norm minimization problem in (3) much faster than standard linear programming.

Natural signals are almost never sparse. But, they often have a sparse representation in some other basis. For example, natural images are sparse in Discrete Cosine Transform (DCT); medical images are sparse under finite differencing; EEG signals are sparse in the Gabor basis and so on. When the sparsifying transform is orthogonal or tight-frame[1], the following analysis-synthesis equations hold. Here, α is the signal transform coefficient by the sparsity basis, ψ.

$$analysis:\; \alpha = \psi x \tag{4}$$

$$synthesis:\; x = \psi^T \alpha \tag{5}$$

1

$$Orthogonal:\; \psi^T\psi = I = \psi\psi^T$$
$$Tight-frame:\; \psi^T\psi = I \neq \psi\psi^T$$

The synthesis equation can be incorporated into (1) as shown below,

$$y = A\psi^T\alpha \tag{6}$$

Since α is sparse, it can be recovered via l_1-minimization (3) and x can be reconstructed by applying the synthesis equation.

In order to guarantee recovery, compressed sensing requires that the measurement basis, A, and the sparsity basis, ψ, be maximally incoherent from each other [17]. Incoherence is maximum between the Dirac/Identity basis and the Fourier basis. Other measurement basis like i.i.d Gaussian matrices, Bernoulli matrices, random Fourier matrices etc. are also incoherent with most of the sparsifying transforms. For practical problems, the measurement basis is dictated by the physics of the acquisition process. For example, for Magnetic Resonance Imaging (MRI) the data are acquired in the Fourier frequency space and for Computer Tomography, the measurement basis is the radon transform. For reconstruction, we must choose a sparsity basis such that: (1) the signal is sparse in the basis and (2) the sparsity basis is incoherent with the measurement basis.

III. Experimental Evaluation of Compressed Sensing with Array Processing

We test our proposed solution on a phantom radar target that is simulated in Matlab 2012a. The target is $0.5m \times 1m$ size and is moving at a velocity of 1.25m/s from an initial standoff distance of 10m towards the radar over 2 seconds. 20×20 sensors are uniformly spaced at 0.5λ to form a $10\lambda \times 10\lambda$ two-dimensional antenna aperture where the wavelength of the radar, λ, is equal to 4cm. The data are captured at a sampling frequency of 400Hz.

Data acquisition for two-dimensional (2D) array processing can be modeled as:

$$y = Fx \tag{7}$$

where x is the image (to be reconstructed), F is the Fourier transform and y are the measured Fourier coefficients along the sensors of the radar. Since the target is moving, we acquire a video sequence of radar imaging data rather than a single frame. Therefore instead of (7), we have a data acquisition model

$$y_t = Fx_t,\, t = 1, ..., T \tag{8}$$

where t denotes the t^{th} frame and T is the total number of frames. If the data are collected on a uniform Cartesian grid, the image of the target can be reconstructed by applying inverse 2D FFT on the acquired data. This is represented as:

$$\hat{x}_t^{ref} = F^T y_t \tag{9}$$

Fig.1 shows one frame of the image of the phantom target that is generated by the application of 2D array processing (9) on the radar data. The image is a function of the sine of the azimuth and elevation positions from the radar. A 2D window function was applied on y_t which has resulted in the sidelobes that are observed in the radar image. The image has 30dB dynamic range. Placing sensors at every location of the

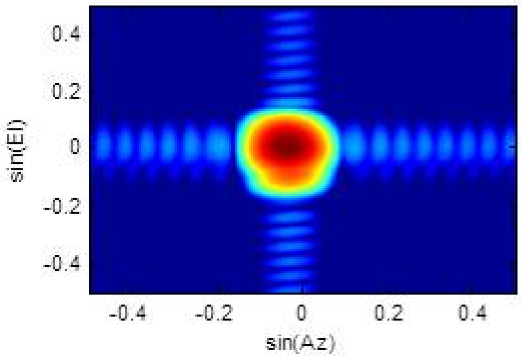


Fig. 1. Image of phantom radar target generated by 2D array processing with 20 x 20 sensors

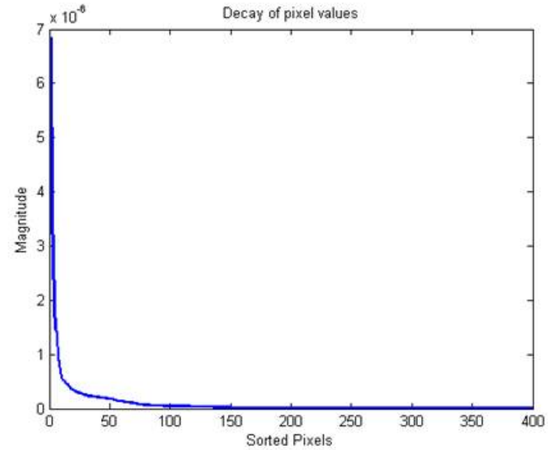


Fig. 2. Decay of pixel values of a single frame

TABLE I
RECONSTRUCTION ERRORS

	50% sampling	25% sampling
Piecemeal	0.1240	0.2824
KCS	0.1133	0.2564

Cartesian grid is costly. Therefore, the challenge here is to reduce the number of required sensors for the given aperture size. This is possible only if the sensor grid space is partially sampled. Such a data acquisition can be modeled as:

$$y_t = RFx_t \tag{10}$$

where R is the sub-sampling mask i.e. it has 1's at sampling locations and 0's at unsampled locations. Therefore, the objective is to reconstruct the image from the partially sampled coefficients so that the reconstructed image (say $\hat{x}_t^{rec}$) is as close as possible to the image acquired from the fully sampled data ($\hat{x}_t^{ref}$). In other words, we must solve an under-determined inverse problem using compressed sensing.

A. Piecemeal Reconstruction

First, we propose a solution which reconstructs each frame individually. CS demands that sampling masks, R, be random in nature [18]. Therefore the positions of the sensors are chosen randomly at $t = 0$ and subsequently fixed for all the frames. Since the measurement basis is a random partial Fourier basis, we choose to reconstruct the image using the Dirac basis as the sparsity basis. The Dirac basis is chosen because of two factors. First, the basis is maximally incoherent with the measurement basis [17], [18]. Second, the radar image is sparse in the spatial domain, i.e. it is sparse in the Dirac basis. Fig.2 shows the sorted absolute pixel values of one frame. The decay is very fast, implying that our sparsity assumption holds. We recover each of the frames, individually, by solving the following l_1-norm minimization problem,

$$\hat{x}_t = \min_x \|x_t\|_1 \; subject\, to\, y_t = RFx_t \tag{11}$$

Off-the-shelf l_1-minimization solvers did not yield good results; therefore we coded our own solver based on cooling the standard iterative soft thresholding (IST) algorithm [19]. The actual aim is to solve the constrained problem (11). However, since solving it directly is difficult, the cooling method proposes an alternative; it solves a series of unconstrained optimization problems of the form:

$$\min_x \|y_t - RFx_t\| + \lambda \|x_t\|_1 \tag{12}$$

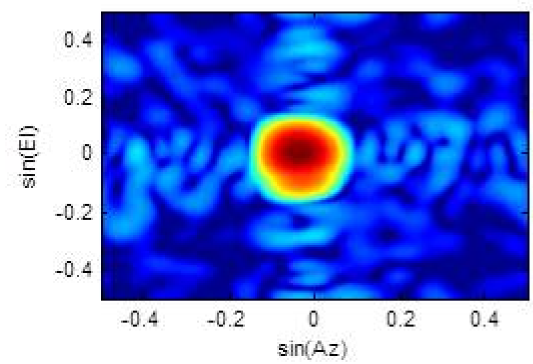
Solving (12) is easy using iterative soft thresholding. The cooling algorithm consists of two loops. The inner loops solves (12) via IST. The outer loops cools the value of λ. The algorithm starts with a high value of λ, but progressively reduces in each outer loop. This technique has been successfully used before for solving such constrained optimization problems [20], [21] and [22]. For this work, we have to start with $\lambda_{init} = \max(abs(F^T R^T y_t))$; in each iteration λ is decreased by 50% and this is continued till the value of λ has fallen to 10^{-4} times its initial value.

We reconstructed the radar image with 50% of the sampling sensors and with 25% of the sampling sensors. Fig. 3 shows the reconstructed images for the same frame for both the cases. These images compare favorably with Fig. 1. The reconstruction error for both cases are presented in Table I. The error is measured in terms of Normalized Mean Squared Error, $NMSE = \frac{\|\hat{x}_t^{ref} - \hat{x}_t^{rec}\|_2}{\|\hat{x}_t^{ref}\|_2}$. This is a standard metric used for verifying CS reconstructions.

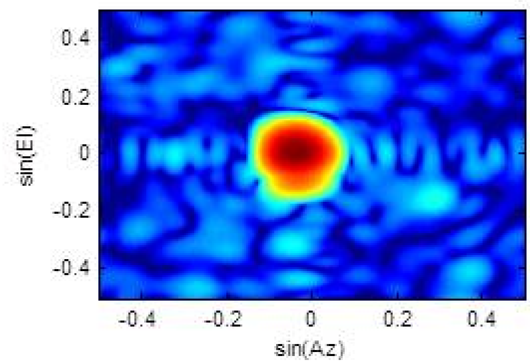
B. Kronecker Compressed Sensing

Frames in a video sequence are temporally correlated because the change between subsequent frames is marginal. In the piecemeal solution, we do not exploit the inter-frame temporal correlation. It has been found that for dynamic MRI reconstruction, exploiting the temporal correlation improves the reconstruction accuracy [23], [24]. In this work, we propose to apply the same technique to improve the accuracy of the radar imaging. The data acquisition model (9) can be succinctly represented as:

$$vec(y) = (I \otimes RF)vec(x) \tag{13}$$



(a)



(b)

Fig. 3. Image of phantom radar target generated by piecemeal compressed sensing in conjunction with array processing with (a) 50% and (b) 25% of total number of sensors used for two-dimensional array processing

where $y = [y_1|\cdots|y_T]$, $x = [x_1|\cdots|x_T]$, I is the identity and vec has the usual connotation. The problem is to reconstruct x_t. We observed that at each pixel location, the temporal variation is sparsely represented in the DCT domain, i.e. if we look at position i in the frame x_t, we will see that the signal $[x_1(i), x_2(i)\cdots, x_T(i)]$ is sparse in DCT. In other words, x is sparse in DCT along the rows. We can represent this sparsity in the following fashion,

$$\alpha = vec(xD) \tag{14}$$

where D is the DCT. This can be conveniently expressed in Kronecker product form as follows,

$$\alpha = (D^T \otimes I)vec(x) \tag{15}$$

where I is the Dirac/Identity basis - the sparsity basis along the columns of x. As DCT is orthogonal, the Kronecker basis $(D^T \otimes I)$ is orthogonal as well. This allows us to express (13) in the following form:

$$vec(y) = (I \otimes RF)(D^T \otimes I)^T \alpha \tag{16}$$

Typically α has a sparser representation compared to the individual frames, and thus yields better reconstruction when solved by l_1-norm minimization. Once (16) is solved via l_1-minimization, we obtain x via,

$$\hat{x} = (D^T \otimes I)^T \alpha \tag{17}$$

Again, we test KCS with 50% sampling and with 25% sampling sensors. The reconstructed images for the same frame are presented in Fig.4. There appears to be reduced error when compared to Fig.3. We present the reconstruction errors in Table I. The KCS yields lower errors when compared to piecemeal reconstruction as anticipated.

IV. Conclusion

Compressed sensing is combined with two-dimensional array processing to reduce the number of sensors required for generating frontal images of moving radar targets. The reconstructed radar images with CS compare favorably with the images generated by conventional array processing for a phantom radar target. Inter-frame correlation across multiple frames is successfully exploited by Kronecker compressed sensing for yielding better results when compared to piecemeal reconstruction.

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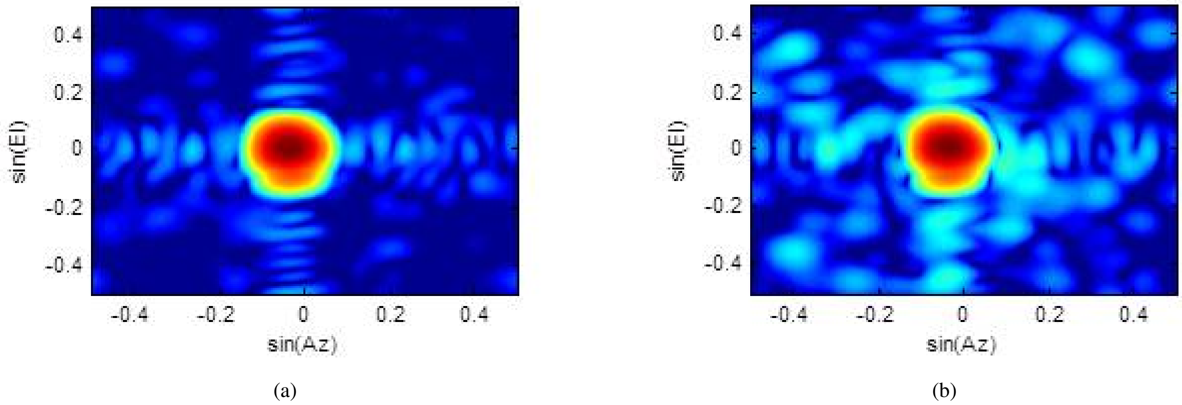

Fig. 4. Image of phantom radar target generated by Kronecker compressed sensing in conjunction with array processing with (a) 50% and (b) 25% of total number of sensors used for two-dimensional array processing

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