

Optimization of Radar Parameters for Maximum Detection Probability Under Generalized Discrete Clutter Conditions Using Stochastic Geometry

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Abstract—We propose an analytical framework based on stochastic geometry (SG) formulations to estimate a radar’s detection performance under generalized discrete clutter conditions. We model the spatial distribution of discrete clutter scatterers as a homogeneous Poisson point process and the radar cross-section of each extended scatterer as a random variable of the Weibull distribution. Using this framework, we derive a metric called the radar detection coverage probability as a function of radar parameters such as transmitted power, system noise temperature and radar bandwidth; and clutter parameters such as clutter density and mean clutter cross-section. We derive the optimum radar bandwidth for maximizing this metric under noisy and cluttered conditions. We also derive the peak transmitted power beyond which there will be no discernible improvement in radar detection performance due to clutter limited conditions. When both transmitted power and bandwidth are fixed, we show how the detection threshold can be optimized for best performance. We experimentally validate the SG results with a hybrid of Monte Carlo and full wave electromagnetic solver based simulations using finite difference time domain (FDTD) techniques.

Index Terms—stochastic geometry, radar detection, FDTD, Monte Carlo simulations, indoor clutter, Poisson point process

I. INTRODUCTION

Clutter models serve as predictive tools for setting detection thresholds and calibrating a radar’s performance during operation [1]. Empirical models of clutter from land terrains, sea and precipitation have been generated, over the decades, using large volumes of high quality measurement data [2]–[4]. These data are typically laborious and time consuming to gather. Further, the computation of the statistical properties - such as higher order moments - from the data may be challenging [5], [6]. An alternate approach would be to use computational tools to model clutter returns [7], [8]. Full wave electromagnetic solvers are deterministic and computationally expensive especially at microwave and millimeter wave frequencies. Reliable characterization of radar detection metrics would require extensive trials of full wave simulations to capture the stochastic nature of the target and clutter conditions. In particular, we need to consider multiple target and clutter deployments and, for each of such realization, several clutter field-of-view and cross-section instances must be considered. Moreover, this procedure would have to be repeated for every set of possible radar parameters, e.g., bandwidth and center frequency. Therefore, such system level simulations are rarely used to quantify radar detection characteristics. In our paper, we propose the use of stochastic geometry (SG) to provide analytical expressions to quantitatively estimate a radar’s detection performance.

In recent years, SG has been explored for a variety of applications [9]. For example, SG has been extensively studied in the context of different types of communication systems including cellular networks [10], [11], millimeter wave systems [12], [13], MIMO systems [14], [15], and vehicular networks [16]. The framework has been exploited for optimizing communication parameters such as data rates

under the given signal to interference ratio when multiple transmitters (such as base stations) and receivers (mobile end users) coexist in the channel. Such an analytical characterization is especially useful in capturing the diversity in the distribution and strength of the base station signals in the region of interest.

More recently, SG-based analysis has been extended to radar scenarios [17]–[21]. In [17], [21], the authors modeled the distribution of the vehicles, mounted with automotive radars, on the roads as a Poisson point process, while in [18], [20], the distribution of pulsed-radar sensors were modeled as a Poisson point process. In all of the above works, SG techniques are used to characterize the interference characteristics from multiple radars and analytically derive the signal to interference and noise ratio (SINR). The radar detection performance was then quantified on the basis of whether the SINR was above a pre-determined threshold. In this paper, we model the locations of discrete clutter scatterers as a Poisson point process similar to [22]. Then, by employing tools from SG, we derive the signal to clutter and noise ratio (SCNR) in the region of interest. The radar detection performance is subsequently quantified based on whether the SCNR exceeds a pre-determined threshold. Note that this formulation of the radar detection performance (and those of the prior SG-based radar works) is fundamentally distinct from classical radar detection frameworks, such as the Neyman-Pearson approach, where likelihood ratio tests between alternative and null hypotheses are performed to generate radar operating curves. In classical radar detection theory, the objective is usually to derive the probability density functions (pdf) for signal plus noise and clutter (or interference) - the alternative hypothesis - and just noise and clutter (or interference) - the null hypothesis [23]. In contrast, our SG-based radar work, characterizes the pdf of the SCNR directly. The main advantage is that SG-based analysis characterizes the radar performance for all possible spatial configurations of network entities (in this case, discrete clutter scatterers) without the need for extensive simulations. Additionally, under suitable assumptions, such an analysis leads to tractable analytical results which enables the derivation of system-design insights that are often missed by system-level simulations [24].

Two typical scenarios where discrete clutter are encountered are indoor radars [25] and foliage penetration radars [26]. Both these radars typically operate at carrier frequencies below the X-band in order to facilitate non-line-of-sight (NLOS) spotting of humans. Indoor radars encounter significant discrete clutter scatterers such as furniture, walls, ceilings and floors [27]. Each of these discrete scatterers consist of multiple scattering centers with significant aspect angle variation. Many different algorithms and strategies have been proposed for mitigating both static and dynamic indoor clutter [28]–[31]. All of these studies focus on specific room layouts and wall geometries. However, across different indoor environments, the scatterers may show considerable variations in their spatial distributions and sizes. Hence these radars may encounter considerable variations in the clutter returns during actual deployments. Similarly, in foliage penetration radar, returns from trees give rise to significant clutter

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[26]. Again, there can be considerable spatial randomness in the tree's girth and distribution. In both the cases, the radar performance is based on the clutter characteristics such as its distribution, the radar cross-section (RCS) of each clutter scatterer as well as the interference between the scatterers. Similarly, variations in the target RCS and position can also affect the radar detection performance. The radar operating curves, in these scenarios, is directly a function of the SCNR which is a random function of both target and clutter parameters. In our preliminary work in [32], we proposed a metric termed as the radar detection coverage probability (P_{DC}) that approximates the fraction of the locations of a target in the region of interest where SCNR is above a predefined threshold. In that paper, we assumed an exponential distribution for the RCS of the discrete clutter and derived P_{DC} for both line-of-sight (LOS) and NLOS conditions of the target. Radar clutter have been modeled using Rayleigh, log-normal or Weibull distributions in prior works [33]–[35]. The log-normal model is typically used when the radar sees land clutter or sea clutter at low grazing angles, while the Rayleigh model is used when the amplitude probability distribution of the clutter is of a limited dynamic range. However, both of these models provide limiting distributions for most experimentally measured clutter. The Weibull model, on the other hand, provides a much more generalized model for radar clutter and is adopted in this work to model the returns from extended scatterers. We also assume that the noise, target and clutter statistics do not change appreciably during the coherent processing interval of the radar. These conditions are generally met for microwave or millimeter-wave radars. Our contributions in this paper are summarised below:

- 1) We model discrete clutter scatterers as a homogeneous Poisson point process. We provide a comprehensive analysis of P_{DC} under generalized clutter conditions modeled using the Weibull function. Here, the exponential and Rayleigh clutter models are just two specific cases of the generalized Weibull clutter model. P_{DC} also considers diversity in the RCS of the radar target, the path loss function, the clutter density and the mean RCS of the clutter scatterers. The theorems, we offer, provide physics based insights into the radar performance with respect to different radar, target and clutter parameters instead of laborious measurement experiments and computationally complex simulation studies.
- 2) The clutter returns that significantly affect the system performance arise from the same range cell occupied by the target. Greater bandwidth giving rise to small range cells result in weaker clutter returns. Noise, on the other hand, increases proportionately to bandwidth. We provide a tractable method for optimizing the radar bandwidth for maximizing the radar detection coverage probability under discrete clutter and Gaussian noise conditions. The optimum radar bandwidth, derived from our theorem, will increase the likelihood of a target being detected across the entire region of interest.
- 3) Under noise limited conditions, increase in transmitted power results in improved radar detection. However, in clutter limited scenarios, there is no further improvement in the radar's performance due to increased returns from the clutter scatterers. We provide an analytical solution for optimizing the radar transmitted power. When both the radar power and bandwidth are fixed, we provide a method to adjust the detection threshold.
- 4) Finally, all of the existing SG works have validated the results using Monte Carlo simulations. In this work, we use a hybrid of finite difference time domain (FDTD) techniques based on full wave electromagnetic solvers and Monte Carlo simulations to validate our SG results. To the best of our knowledge, our

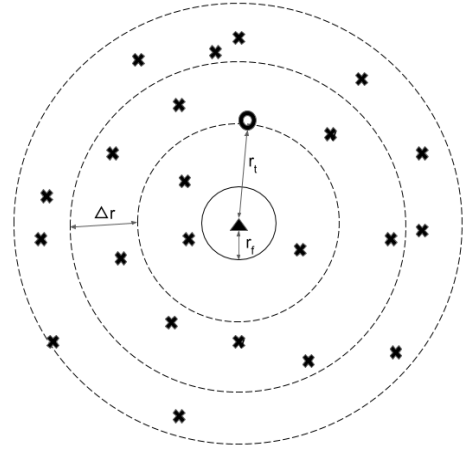


Fig. 1: In the stochastic geometry formulation shown above, the monostatic radar (indicated as a triangle) is at the center of the region of interest with a single target (indicated as a circle) at a distance r_t from the radar. Discrete clutter scatterers (indicated as crosses) are modelled as a homogeneous Poisson point process (PPP). They are distributed uniformly in the region of interest except within the far-field radius of the radar (r_f).

work is the first to present an experimental validation of SG results with actual electromagnetic modeling.

The paper is organized as follows. In Section II, we present two theorems. The first theorem derives the P_{DC} using SG formulations while the second theorem derives the optimum radar bandwidth for maximizing P_{DC} . In the following Section III, we describe the experimental validation using a hybrid of FDTD and Monte Carlo simulations. Finally, in Section IV, we present the key results and analyses regarding the radar detection performance as a function of radar parameters - such as radar bandwidth, transmitted power and receiver system noise - target parameters (target position, mean target RCS) and clutter parameters (mean clutter RCS, clutter density and type of distribution). The scientific inferences from our studies are summarized in the last section.

Notation We use the following notation in the paper. We denote random variables by upper-case letters and their realizations or other deterministic quantities by lower-case letters. We use the bold font to denote vectors and normal font to denote scalar quantities.

II. THEORY

Consider a monostatic radar of wavelength λ_c located at the origin of a two dimensional circular space and shown as a triangle in Fig. 1. The transmitted power from the radar is P_{tx} . We assume that the radar consists of directional transmitting and receiving antennas of gain $G_{tx}(\theta)$ and $G_{rx}(\theta)$ respectively where θ is the direction-of-arrival of a direct path signal from a point scatterer (target or clutter). Noise at the radar receiver is assumed to be white Gaussian and is given by $N_s = K_B T_s BW$ where K_B is the Boltzmann constant, T_s is the system noise temperature and BW is the radar bandwidth. The bandwidth of the radar also determines the radar range resolution cell size given by $\Delta r = \frac{c}{2BW}$ where c is the speed of light.

A single target, indicated as a circle in the figure, is assumed to be present in the channel at a fixed position $x_t = (r_t, \theta_t)$. Here, the target's Euclidean distance from the radar, r_t , can range from $r_t \in (r_f, \infty]$ where r_f is the Fraunhofer far-field distance of the radar antennas. The target azimuth, θ_t , is fixed such that the target is assumed to be within the main lobe of the radar where $G(\theta_t) =$

$G_{tx}(\theta_t)G_{rx}(\theta_t) = 1$. The target radar cross-section (σ_t) is a random variable described by the Swerling 1 model with average RCS of $\sigma_{t,avg}$ as shown in

$$P(\sigma_t) = \frac{1}{\sigma_{t,avg}} \exp\left(\frac{-\sigma_t}{\sigma_{t,avg}}\right). \quad (1)$$

This corresponds to the case of a target composed of several scatterers of approximately equal reflectivities (like a human). Based on the radar range equation, the received signal at the radar for a single pulse is:

$$S(r_t) = P_{tx}\sigma_t \mathcal{H}(r_t). \quad (2)$$

Here $\mathcal{H}(r_t)$ is the path loss function which is a function of the distance between the radar and target as shown below

$$\mathcal{H}(r_t) = PL_f \left(\frac{r_f}{r_t}\right)^{2q}, \quad (3)$$

where q is the path loss coefficient and PL_f is the path loss factor at r_f .

Next, we discuss the clutter characteristics. We assume that there are discrete point scatterers that constitute the clutter (shown as crosses in the figure). We also assume that the coherent processing interval of the radar is short compared to the time required for the clutter statistics to change. These conditions are generally met for microwave or millimeter-wave radars. These clutter point scatterers are randomly distributed in the two-dimensional space. The locations of the random clutter scatterers are modeled as a homogeneous Poisson point process (PPP), Φ . The number of scatterers in a closed compact set A is denoted by the intensity measure $\rho\nu(A)$ where ρ is the clutter density and $\nu(A)$ is the size of the illuminated area, A . Each realization of Φ is denoted by ϕ and the location of each clutter point is a random vector, $\mathbf{X}_c = (R_c, \theta_c)$. Again, the clutter points are assumed to be a minimum far-field distance r_f away from the radar. The number of point clutter in each realization follows the Poisson's distribution while their distribution is assumed to be uniform within the region of interest from $[r_f, \infty)$ and $0 \leq \theta_c \leq 2\pi$. However, we only focus on the returns from those clutter points that lie within the same range resolution cell as the target (when $r_t - \frac{\Delta r}{2} < R_c < r_t + \frac{\Delta r}{2}$) since these are the returns that are most likely to impact the detection. Therefore the mean number of clutter scatterers within this range cell is given by $\rho 2\pi r_t \Delta r$. The radar signal reaching each clutter scatterer is affected by the path loss function based on a slow fading model denoted as $\mathcal{H}(R_c)$. Each discrete clutter scatterer is considered an extended scatterer and modelled to have a fluctuating RCS (σ_c) based on the Weibull model with an average RCS of $\sigma_{c,avg}$, as shown in

$$P(\sigma_c) = \frac{\alpha}{\sigma_{c,avg}} \left(\frac{\sigma_c}{\sigma_{c,avg}}\right)^{\alpha-1} \exp\left(-\left(\frac{\sigma_c}{\sigma_{c,avg}}\right)^\alpha\right), \quad (4)$$

where α is the shape parameter. When $\alpha = 1$ the above expression reduces to the exponential probability distribution (4). When $\alpha = 2$, the above expression shows the Rayleigh probability distribution. Thus the total clutter returns at the radar receiver, for each realization of the PPP, depends on the clutter points within the range resolution cell of the target and is given by

$$C = \sum_{c \in \phi, c \in r_t - \frac{\Delta r}{2}, r_t + \frac{\Delta r}{2}} P_{tx}G(\theta_c)G_c\sigma_c \mathcal{H}(R_c). \quad (5)$$

In the above equation, the interference between the clutter returns from the individual point scatterers is captured by G_c .

There can be significant variations in the clutter returns due to variation in the number, the distribution and fluctuations in RCS of the extended clutter scatterer. Similarly, the target returns may vary

due to fluctuations in the target RCS. Therefore, the *mean* signal to clutter and noise ratio at the radar for a target at a given r_t is

$$SCNR(r_t) = \mathbb{E}_{\sigma_c, G_c, \Phi} \left[\frac{P_{tx}\sigma_t \mathcal{H}(r_t)}{\sum_{c \in \phi} P_{tx}G(\theta_c)G_c\sigma_c \mathcal{H}(R_c) + N_s} \right]. \quad (6)$$

Classical radar detection theory considers the radar operating curves derived from the probability of detection (P_D) and probability of false alarm (P_{FA}). There are many works in literature that have proposed approximations to the relationships between P_D , P_{FA} and SNR. However, as the scenario becomes more complex, with a large number of discrete clutter scatterers with considerable variation in their spatial distribution and cross-sections, the relationship between P_D , P_{FA} and SCNR becomes harder to derive analytically. Instead, we propose an alternative and simpler metric called the *radar detection coverage probability* (P_{DC}) based on the mean SCNR. Thus P_{DC} is distinct from both P_D and P_{FA} used in classical radar and indirectly includes the effect of both detections and false alarms. The metric is analogous to the *cellular coverage probability* in wireless communications. There the metric is defined as the probability that a mobile user at any particular position in the coverage area will experience a signal to interference and noise ratio above a predefined threshold. The metric provides a method for evaluating the network performance while trading off between the benefits of increased capacity with greater density of mobile base stations with the performance issues that arise due to interference between the base stations. The metric also provides a method for optimizing some of the cellular parameters (such as data rate) for a given SINR. In the case of radar, greater bandwidth results in a smaller range resolution cell. If we consider the clutter that arises from the same cell as the target, then a smaller range cell results in reduced clutter. However, higher bandwidth also results in greater system noise at the radar receiver. Therefore, we propose to use the metric P_{DC} to optimize the radar bandwidth with respect to noise and clutter conditions.

We first define the P_{DC} metric below and provide a analytical framework for deriving it based on radar, target and clutter conditions.

Definition 1. *The radar detection coverage probability (P_{DC}) is defined as the probability that the SCNR for a single target at a Euclidean distance r_t from a monostatic radar, is above a predefined threshold γ : $P_{DC}(r_t) \triangleq \mathbb{P}(SCNR(r_t) \geq \gamma)$.*

Theorem 1. *The radar detection coverage probability of a target at a distance r_t from the radar is given by*

$$P_{DC}(r_t) = \exp\left(\frac{-N_s\gamma}{P_{tx}\mathcal{H}(r_t)\sigma_{t,avg}}\right) \dots \exp(-\rho r_t \Delta r \int_{\phi_c} (1 - J(\theta_c)) d\phi_c), \quad (7)$$

where

$$J(\theta_c) = \int_0^\infty \exp\left(\frac{-\gamma G(\theta_c)\sigma_c}{\sigma_{t,avg}}\right) P(\sigma_c) d\sigma_c. \quad (8)$$

Proof. From (6), we can see that the P_{DC} at a given r_t can be written in terms of the target cross-section in

$$P_{DC}(r_t) = \mathbb{P}\left[\sigma_t > \frac{N_s\gamma}{P_{tx}\mathcal{H}(r_t)} + \frac{\gamma}{\mathcal{H}(r_t)} \sum_{c \in \phi} G(\theta_c)G_c\sigma_c \mathcal{H}(R_c)\right]. \quad (9)$$

Due to the exponential distribution of the target RCS in (1), the above expression becomes

$$\begin{aligned} \mathbb{E}_{\sigma_c, G_c, \Phi} \left[\exp \left(\frac{-N_s \gamma}{P_{tx} \mathcal{H}(r_t) \sigma_{t_{avg}}} + \sum_{c \in \phi} \frac{-\gamma G(\theta_c) G_c \sigma_c \mathcal{H}(R_c)}{\mathcal{H}(r_t) \sigma_{t_{avg}}} \right) \right] \\ = \exp \left(\frac{-N_s \gamma}{P_{tx} \mathcal{H}(r_t) \sigma_{t_{avg}}} \right) I(\cdot) \end{aligned} \quad (10)$$

In the above expression, the terms within the first exponent - N_s , γ , P_{tx} , $\mathcal{H}(r_t)$ and $\sigma_{t_{avg}}$ - are all deterministic. This term encompasses the signal-to-noise (SNR) ratio with respect to the target returns. The second exponential term, $I(\cdot)$ shows the stochasticity of the clutter conditions (clutter cross-section and the PPP distribution of the clutter points). This term is independent of P_{tx} and is a function of the signal-to-clutter ratio (SCR) of the system. $I(\cdot)$ can be further evaluated as

$$I(\cdot) = \mathbb{E}_{\sigma_c, G_c, \Phi} \left[\prod_{c \in \Phi} \exp \left(\frac{-\gamma G(\theta_c) G_c \sigma_c \mathcal{H}(R_c)}{\mathcal{H}(r_t) \sigma_{t_{avg}}} \right) \right], \quad (11)$$

since the exponential of sum terms can be written as the product of exponential terms. Using the probability generating functional (PGFL) of SG [36], we obtain

$$\begin{aligned} I(\cdot) &= \exp(-\rho \int_{r_t - \frac{\Delta r}{2}}^{r_t + \frac{\Delta r}{2}} \int_0^{2\pi} (1 - \dots \\ &\mathbb{E}_{\sigma_c, G_c} \left[\exp \left(\frac{-\gamma G(\theta_c) G_c \sigma_c \mathcal{H}(r_c)}{\mathcal{H}(r_t) \sigma_{t_{avg}}} \right) \right] d(\vec{x}_c)) \\ &= \exp \left(-\rho \int_{r_t - \frac{\Delta r}{2}}^{r_t + \frac{\Delta r}{2}} \int_0^{2\pi} (1 - J(\cdot)) r_c d\phi_c dr_c \right) \end{aligned} \quad (12)$$

As pointed out before, we only consider the discrete clutter that arise within the same range cell as the target. The inner expectation term, $J(\cdot)$, depends on the distribution of σ_c and the interference term G_c . Under worst case scenarios, G_c is always 1 and the clutter returns add. Then

$$\begin{aligned} J(\cdot) &= \mathbb{E}_{\sigma_c} \left[\exp \left(\frac{-\gamma G(\theta_c) \sigma_c \mathcal{H}(r_c)}{\mathcal{H}(r_t) \sigma_{t_{avg}}} \right) \right] \\ &= \int_0^\infty \exp \left(\frac{-\gamma G(\theta_c) \sigma_c \mathcal{H}(r_c)}{\mathcal{H}(r_t) \sigma_{t_{avg}}} \right) P(\sigma_c) d\sigma_c \end{aligned} \quad (13)$$

Now if the range resolution cell is sufficiently narrow, which is usually the case for microwave and millimeter wave radars, then $\mathcal{H}(r_c) \sim \mathcal{H}(r_t)$ when $r_t - \frac{\Delta r}{2} \leq r_c \leq r_t + \frac{\Delta r}{2}$. Therefore (13) becomes independent of r_t as shown below in

$$J(\theta_c) = \int_0^\infty \exp \left(\frac{-\gamma G(\theta_c) \sigma_c}{\sigma_{t_{avg}}} \right) P(\sigma_c) d\sigma_c. \quad (14)$$

Substituting this in (12), we obtain

$$I = \exp(-\rho r_t \Delta r \int_{\phi_c} (1 - J(\theta_c)) d\phi_c). \quad (15)$$

Combining (13), (11) and (10), we prove the theorem. \square

In the above discussion not all distributions lead to tractable solutions. For example, the choice of exponential model (Swierling 1/2) model of the target RCS was crucial. The higher order Swierling 3 distribution results in far more challenging mathematical operations and hence not discussed here. In [32], we discussed the effect of the gain of the radar antennas on the detection performance. We do not repeat that discussion here and confine our discussion to isotropic radar antennas where $G(\theta_c) = 1$. The $J(\cdot)$ term which is a function of the clutter cross-section can be computed numerically. However

for two cases, when $\alpha = 1$ corresponding to exponential distribution and for $\alpha = 2$ corresponding to Rayleigh distribution, analytical expressions for $J(\cdot)$ are derived.

Case 1: When $\alpha = 1$ for exponential distribution of clutter, with mean clutter cross-section, $\sigma_{c_{avg}}$, the $J(\cdot)$ term reduces to

$$J = \frac{1}{1 + \nu} \quad (16)$$

where $\nu = \frac{\gamma \sigma_{c_{avg}}}{\sigma_{t_{avg}}}$. Substituting this expression back in (11), we obtain

$$I = \exp \left(-2\pi\rho \frac{\nu}{\nu + 1} r_t \Delta r \right) \quad (17)$$

which can be easily evaluated numerically.

Case 2: When $\alpha = 2$ for Rayleigh distribution of clutter, the analytical solution for J is

$$J = 1 - \frac{\sqrt{\pi\nu}}{2} e^{\nu^2/4} \operatorname{erf}\left(\frac{\nu}{2}\right), \quad (18)$$

where $\operatorname{erf}(\cdot)$ is the error function. This results in

$$I = \exp \left(-\pi^{1.5} \rho \nu e^{\nu^2/4} \operatorname{erf}\left(\frac{\nu}{2}\right) r_t \Delta r \right) \quad (19)$$

Corollary 1.1. *In case 1, when ν is much greater than 1 which corresponds to the situation when $\sigma_{c_{avg}} \gg \sigma_{t_{avg}}$, then I becomes independent of ν . As a result, P_{DC} becomes independent of $\sigma_{c_{avg}}$. Similarly, in case 2, the exponential term ($e^{\nu^2/4}$) within J becomes very high when ν is high. As a result I converges to 1 and the P_{DC} is no longer a function of $\sigma_{c_{avg}}$. In other words, P_{DC} deteriorates with increase in $\sigma_{c_{avg}}$ till it asymptotically converges at a limit.*

Corollary 1.2. *The same effect of P_{DC} versus $\sigma_{c_{avg}}$ is observed for generalized Weibull clutter parameter α . If we define $\kappa = \frac{-\gamma G(\theta_c) \mathcal{H}(r_c)}{\mathcal{H}(r_t)}$ in the generalized J in (13), then the first derivative of J with respect to $\sigma_{c_{avg}}$ is given by*

$$\begin{aligned} \frac{dJ}{d\sigma_{c_{avg}}} &= \int_0^\infty \exp(-\kappa\sigma_c) \dots \\ &\left[\frac{\alpha \sigma_c^{\alpha-1}}{\sigma_{c_{avg}}^\alpha} \exp \left(-\left(\frac{\sigma_c}{\sigma_{c_{avg}}} \right)^\alpha \right) \left(\frac{-\alpha}{\sigma_{c_{avg}}^{\alpha+1}} - \alpha \left(\frac{\sigma_c}{\sigma_{c_{avg}}} \right)^{\alpha-1} \right) \right]. \end{aligned} \quad (20)$$

In (20), the terms inside the integral are always negative. Since J is bounded below and a decreasing function of $\sigma_{c_{avg}}$, we can conclude that as $\sigma_{c_{avg}}$ tends to ∞ , J will tend to 0. Hence, for high values of $\sigma_{c_{avg}}$, P_{DC} is independent of $\sigma_{c_{avg}}$ for generalized Weibull clutter conditions.

Corollary 1.3. *In (7), it is evident that the first exponential term indicates the effect of the SNR on the radar detection performance while the second term shows the effect of SCR. Hence increase in the transmitted power improves the radar detection performance while the radar operates in the noise limited scenario but has limited impact on the performance when the radar enters the clutter limited scenario. For fixed target and clutter conditions, P_{DC} converges to I_{conv} with increase in P_{tx} . The transmitted power at which detection performance reaches 99% ($e^{-0.01}$) of the convergence value is given by*

$$\begin{aligned} I_{conv} e^{-0.01} &= \exp \left(\frac{-N_s \gamma}{P_{tx} \mathcal{H}(r_t) \sigma_{t_{avg}}} \right) I_{conv} \\ \Rightarrow P_{tx}^{max} &= \frac{100 N_s \gamma}{\mathcal{H}(r_t) \sigma_{t_{avg}}} \end{aligned} \quad (21)$$

The maximum transmitted power is therefore independent of clutter parameters such as ρ , $\sigma_{c_{avg}}$ and α .

Corollary 1.4. In the above discussion, the radar detection coverage probability is provided in terms of the clutter density, ρ . However, in some experiments, other spatial statistical parameters besides clutter density may be used. One popular parameter is the average nearest neighbor distance, \bar{r} , which is the average distance between the centroid of a clutter point and its nearest neighboring clutter point. For a uniform random distribution of the clutter points, the average nearest neighbor distance is related to the clutter density [37] as shown below

$$\bar{r} = \frac{1}{2\sqrt{\rho}} \quad (22)$$

Therefore, the radar detection probability in (7) can be written as

$$P_{DC}(r_t) = \exp\left(\frac{-N_s \gamma}{P_{tx} \mathcal{H}(r_t) \sigma_{t_{avg}}}\right) \cdots \exp\left(-\frac{r_t \Delta r}{2\bar{r}^2} \int_{\phi_c} (1 - J(\theta_c)) d\phi_c\right) \quad (23)$$

When the distribution of the clutter points deviate from the uniform distribution and cluster in some regions, the inhomogeneity of the clutter distribution can be modeled either with the a non-uniform clutter density ($\rho(x_c)$) or through additional statistics on the average nearest neighbor distance [37].

Next, we provide a theorem for optimizing the radar BW under noisy and cluttered conditions.

Theorem 2. For a narrow range resolution cell, the optimum BW for detecting a Swerling 1 target is given by

$$BW = \sqrt{\frac{\pi \rho c (1 - J) r_t P_{tx} \mathcal{H}(r_t) \sigma_{t_{avg}}}{K_B T_s \gamma}}. \quad (24)$$

Proof. For a narrow Δr and isotropic radar antennas, the integral in (15) can be reduced to

$$I(\cdot) = \exp(-2\pi\rho(1 - J)r_t\Delta r) \quad (25)$$

Therefore the radar detection probability is a function of BW as shown in

$$P_{DC} = \exp\left(\frac{-K_B T_s BW \gamma}{P_{tx} \mathcal{H}(r_t) \sigma_{t_{avg}}}\right) \exp\left(-2\pi\rho(1 - J)r_t \frac{c}{2BW}\right). \quad (26)$$

Higher BW results in greater noise which causes a reduction in the P_{DC} due to the first exponential term. However increase in BW also results in a reduced Δr resulting in lesser clutter returns as seen in the second exponential term. Therefore, the optimum BW for maximizing P_{DC} can be determined by taking a natural logarithm on both sides of (26) as shown below

$$\ln P_{DC} = \frac{-K_B T_s BW \gamma}{P_{tx} \mathcal{H}(r_t) \sigma_{t_{avg}}} - 2\pi\rho(1 - J)r_t \frac{c}{2BW}. \quad (27)$$

The optimum BW , shown in (24), is obtained when the first derivative of the above expression with respect to BW is equated to zero. \square

The optimum BW is shown to be a function of the target distance from the radar. In many situations it may not possible to change the radar bandwidth while tracking the target. In those scenarios, it may be preferable to be able to adjust the threshold γ for a fixed transmitted power and radar bandwidth BW .

Corollary 2.1. The γ for obtaining the maximum P_{DC} for a given radar transmitted power and bandwidth for large clutter cross-sections should be adjusted based on the target distance r_t using

$$\gamma(r_t) = \frac{\pi \rho c r_t P_{tx} \mathcal{H}(r_t) \sigma_{t_{avg}}}{K_B T_s BW^2}. \quad (28)$$

We have already observed from corollaries 1.1 and 1.2 that J becomes 0 for high $\sigma_{c_{avg}}$. Therefore, in those scenarios, the above expression is directly obtained from (24). Note that in the above expression, the γ is independent of the type of clutter cross-section distribution (α) and is only dependent on the density of the clutter scatterers.

III. EXPERIMENTAL VALIDATION

In prior works, the experimental validation of the SG results were based on Monte Carlo simulations. However, the complete electromagnetic phenomenology (attenuation, diffraction, scattering) are not captured through these simulations. Therefore, in this paper, we use a full wave electromagnetic solver to model the complete radar propagation phenomenology. However, the computational complexity of these solvers is dependent on the size of the region of interest and the wavelength of the source excitation. Also, these solvers are inherently deterministic and cannot capture the diversity in radar, target and clutter parameters. Therefore, we use a hybrid of electromagnetic based modeling based on finite FDTD and Monte Carlo based simulations to experimentally validate the SG results.

A. Finite Difference Time Domain Simulations

The FDTD technique models the complete propagation physics between a source and the scatterers in a medium. Many prior works have used FDTD for modeling indoor clutter [31], [38]–[42]. For computational simplicity, we consider a two-dimensional (2D) simulation space along the XY plane spanning 20m by 20m. A narrowband sinusoidal infinitely long, line source at 1 GHz emulating the monostatic radar is introduced at the center of the space at (0, 0)m. The space is discretized into grid points that are uniformly spaced $\frac{\lambda_c}{10}$ apart where λ_c is the wavelength corresponding to the source excitation. The entire simulation space is bounded by a perfectly matched layer (PML) of $2\lambda_c$ thickness.

The distribution of discrete clutter scatterers in the FDTD space follows the homogeneous PPP. For a given clutter density ρ , we perform multiple FDTD simulations as shown in Fig.2. Each column in the figure shows three FDTD simulations for a specific ρ . The number of scatterers in a FDTD realization follows the Poisson distribution where the mean number of scatterers across multiple FDTD simulations is $\rho \times A$ where $A = 400 m^2$ is the area of the simulation space. In each FDTD simulation, the scatterers are distributed uniformly about the simulation space except within a radius of $r_f = 3m$ around the source. Each scatterer is modelled as an infinitely long dielectric cylinder of dielectric constant $\epsilon_r = 7.1$ and radius r_g . The RCS, σ_c , of each of the clutter scatterers is a random variable drawn from the Weibull distribution of mean cross-section, $\sigma_{c_{avg}} = 0.8 m^2$ and shape parameter $\alpha = 1$ (or 2) as given in (4). Using modal analysis, the 2D RCS of an infinitely long cylinder is given in terms of Bessel and Hankel functions as shown in

$$\sigma_c = \frac{16}{k} \left[-\frac{\mathcal{J}_0(kr_g)}{\mathcal{H}_0^1(kr_g)} + \sum_{n=1}^N 2(-1)^{n+1} \frac{\mathcal{J}_n(kr_g)}{\mathcal{H}_n^1(kr_g)} \right]^2, \quad (29)$$

where $k = 2\pi\sqrt{\epsilon_r}/\lambda$ and N is the number of modes [43]. The unit of the 2D RCS is in meters rather than square-meters (3D). Based on the above equation, a look up table is formed between r_g and σ_c using a sufficiently large value of N (50, in our case) when σ_c has converged. Using this look up table, r_g is estimated for each σ_c of the scatterer. We further approximate the cylinders to have a square shaped longitudinal cross-section for simplicity.

The FDTD models the time-domain transverse electric field, $E_z(\vec{r}, t)$, in the two-dimensional grid space (\vec{r}). Through Fourier

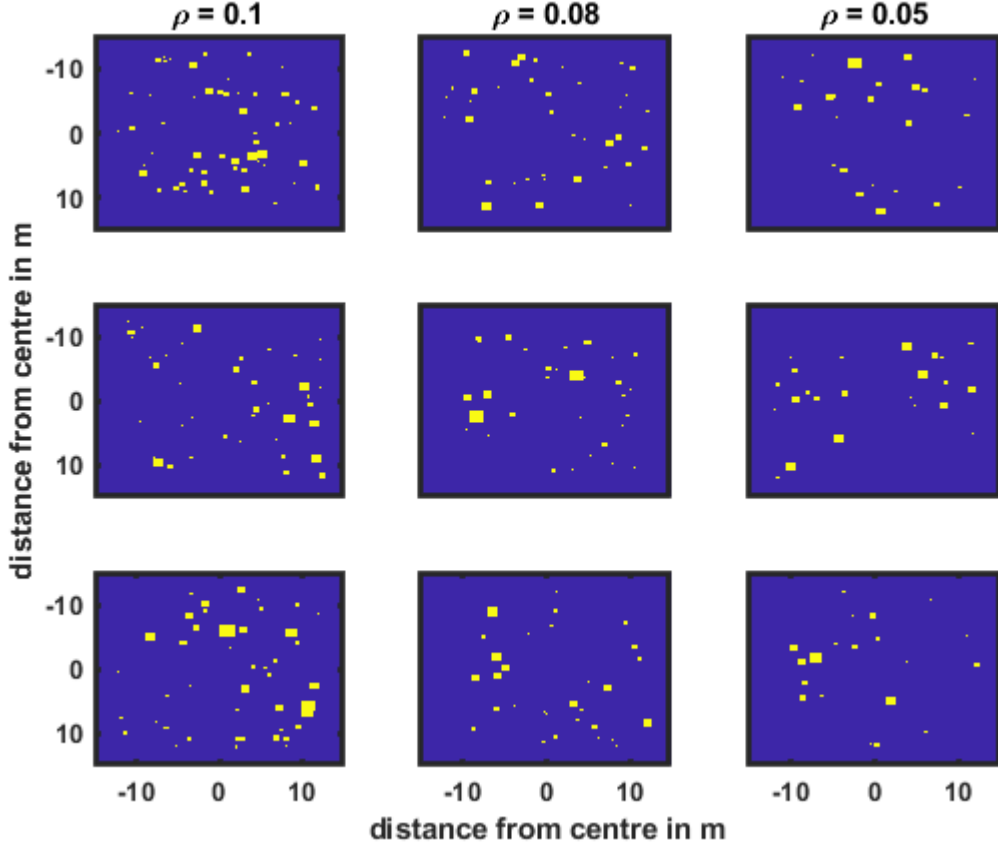


Fig. 2: Two-dimensional FDTD simulation space (20×20 m) with point clutter scatterers of $\epsilon_r = 7.1$ and mean RCS $\sigma_{c_{avg}} = 0.8 \text{ m}^2$ for three different clutter densities: first, second and third columns correspond to $\rho = 0.1, 0.08, 0.05 \text{ 1/m}^2$ respectively. The clutter scatterers are distributed uniformly in the simulation space while the RCS of the clutter scatterers follows the Weibull distribution with $\alpha = 1$.

transform, we obtain the corresponding electric field in frequency domain, $E_z(\vec{r}, f_c)$ for $f_c = 1$ GHz. A second FDTD simulation is run in free space conditions without the presence of any of the scatterers using the same source excitation. Again, the resulting time-domain free space electric field is Fourier transformed to obtain the corresponding frequency-domain response at all the grid positions ($E_z^{fs}(\vec{r}, f_c)$). Then, the two-way path loss, $\mathcal{H}(r)$, at a distance r from the source is obtained from the ratio of the mean of the square of electric field at a distance r from the source and the mean of the square of the corresponding electric field obtained in free space conditions, as shown in

$$\mathcal{H}(r) = \frac{\lambda_c}{(2\pi)^3 r^2} \left[\frac{\oint_{\phi=0}^{2\pi} |E_z(\vec{r}, f_c)|^2 d\phi}{\oint_{\phi=0}^{2\pi} |E_z^{fs}(\vec{r}, f_c)|^2 d\phi} \right]^2. \quad (30)$$

The denominator term essentially normalizes the source excitation in the path loss factor. The power factor of 4 in (30) accounts for the two-way propagation path and includes the effects of propagation through dielectric scatterers, diffraction about the edges of the scatterers and multipath reflections. Note that the path loss factor estimated from this simulation study corresponds to 2D cylindrical waves rather than 3D spherical waves that correspond to the usual radar scenario. Hence, the path loss decays at a rate of $1/m$ (2D) rather than $1/m^2$ (3D). However, in our SG formulations, we are only concerned with the relative power decay from r_f to r and hence the difference in the phase front in 2D and 3D scenarios is ignored. In (30), we estimated $\mathcal{H}(r)$ from the mean power decay corresponding to a circle of radius

r around the source. This is further averaged across multiple FDTD simulations to obtain a fairly good generalized estimate of $\mathcal{H}(r)$ for any given ρ and $\sigma_{c_{avg}}$. We integrate these path loss estimates with Monte Carlo simulations to experimentally validate the stochastic geometry results.

B. Monte Carlo Simulations

The SG results are validated through Monte Carlo simulations. The following parameters are fixed: $P_{tx} = 30$ dBm, $N_s = 300K$, threshold $\gamma = 1$ and carrier frequency of 1 GHz are fixed. The target cross-section, σ_t , in each trial is drawn from the Swerling 1 model in (1) with a mean of $\sigma_{t_{avg}} = 0.8 \text{ m}^2$ while the clutter cross-section of each scatterer is varied based on the Weibull model in (4) with $\alpha = 2$ and $\sigma_{c_{avg}} = 0.8 \text{ m}^2$. We consider three cases of clutter densities: $\rho = 0.1, 0.08, 0.05 \text{ 1/m}^2$. For each case, the number of clutter scatterers in each trial is drawn from the Poisson distribution where the mean is $\rho \times A$ where A is 400 m^2 . We only consider the clutter returns from those that fall within the same range cell as the target. The size of the range cell is determined from the radar bandwidth BW . The path loss factor for both target and clutter scatterers at any position \vec{r} is obtained from the FDTD simulations described above. A total of 10000 trials are conducted to obtain S and C based on (2) and (5) respectively. The mean P_{DC} is computed based on definition 1 and compared with the stochastic geometry solutions obtained from (7). The P_{DC} is plotted for different BW and ρ . The results are presented in Fig.3 for $\alpha = 1$ and $\alpha = 2$. The result shows an exact match between FDTD and SG results for $\alpha = 1$ (Fig.3a) and a very

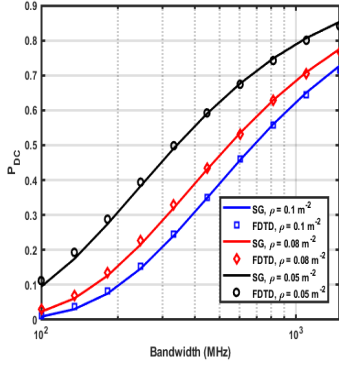
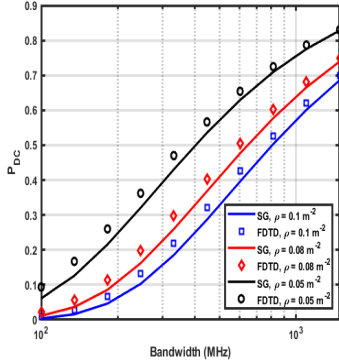
(a) P_{DC} vs. BW, ρ for $\alpha = 1$ (b) P_{DC} vs. BW, ρ for $\alpha = 2$

Fig. 3: FDTD and SG-based estimations of P_{DC} for different BW and ρ when $P_{tx} = 30$ dBm, $N_s = 35$ K, $\sigma_{t_{avg}} = \sigma_{c_{avg}} = 0.8 m^2$, $q = 2$ for SG.

close match when $\alpha = 2$ (Fig. 3b). In each figure, we show the values for the different cases of ρ . The results show that the SG framework is able to accurately estimate the radar's detection performance without the computational complexity of the FDTD framework. Note that the FDTD results have been confined to a region of 20×20 m for a source of 1 GHz. The computation complexity would considerably increase for greater r_t and higher source frequencies due to smaller grid dimensions. The small variation in the results in Fig.3b may be attributed to the fact that the FDTD simulations do not have a circular cross-section as assumed. Second, we have assumed a path loss factor of $q = 2$ in the SG formulations whereas the actual propagation conditions may have a slightly different q due to multipath scattering. Third, the FDTD simulation is inherently deterministic. We estimate the *mean* path loss factor by taking the average of a radius r around the source. We also average across three simulations. However, the resulting estimate of the path loss factor may still not capture the stochasticity inherent in the radar propagation conditions.

IV. RESULTS

In this section, we present the results obtained from the two theorems and their corollaries. The first theorem shows the relationship between the newly proposed metric - radar detection coverage probability - and radar, target and clutter parameters. The second theorem shows the optimum radar bandwidth under both noisy and cluttered conditions. In all of the cases, the radar antennas are assumed to be isotropic ($G_{rx} = G_{tx} = 0$) dBi with a minimum far-field distance of $r_f = 3$ m, while the carrier frequency is fixed at 1 GHz. The mean target cross section, $\sigma_{t_{avg}}$, is assumed to be 1

TABLE I: Radar, Clutter and Target Parameters for Stochastic Geometry Formulations

Radar Parameters		Values
Carrier frequency (f_c)		1 GHz
Transmitted power (P_{tx})		0 to 30 dBm
Noise temperature (N_s)		300 K to 1500 K
Radar bandwidth (BW)		0.1 to 2 GHz
Antenna gain (G)		0 dBi
Target Parameters		Values
Average target RCS ($\sigma_{t_{avg}}$)		1 m^2
Target distance (r_t)		5 to 50m
Clutter Parameters		Values
Average clutter cross-section ($\sigma_{c_{avg}}$)		0.5 to 5 m^2
Weibull clutter shape parameter (α)		1 to 2 (no units)
Clutter density (ρ)		0.003 1/ m^2
Path loss exponent (q)		2 to 4 (no units)
SCNR threshold (γ)		1

m^2 . All the radar parameters - P_{tx} , N_s and BW - can vary and the ranges of their variation are listed in Table.I along with target and clutter parameters.

A. Analysis of Results from Theorem 1

First, we present P_{DC} as a function of target distance (r_t) from the radar for different types of clutter in Fig.4a. For this case, P_{tx} is 30 dBm, N_s is 300 K and BW is 0.1 GHz. Both $\sigma_{c_{avg}} = \sigma_{t_{avg}} = 1m^2$ while $q = 2$. We plot the variation of P_{DC} for different values of Weibull shape parameter, α . When $\alpha = 1$, the RCS of discrete clutter scatterers are of exponential distribution while $\alpha = 2$ corresponds to scatterers with RCS of Rayleigh distribution. We observe that P_{DC} falls with increase in r_t . This is because the increase in path loss term $\mathcal{H}(r_t)$ results in the fall of the SNR (first exponential term) in (7). The SCR in the second exponential term is less impacted by $\mathcal{H}(r_t)$ since the clutter scatterers that fall within the same range cell as the target are similarly impacted by path loss. There is a slight increase in the clutter returns due to increase in clutter area size ($2\pi r_t \Delta r$). The detection performance deteriorates slightly for higher values of α . The exponential distribution provides the upper bound of the performance while the Rayleigh distribution provides the lower bound.

Next, we use the same parameters that were used in the previous case. However, this time α is fixed at 1 while q is varied from 2 to 4 in Fig.4b. Higher values of q incorporate multipath scattering effects into the path loss function $\mathcal{H}(\cdot)$. As expected, higher q results in significant deterioration in the radar detection performance.

Next, we examine the effect of increasing P_{tx} on P_{DC} in Fig.4c for different values of α . Here, the distance of the target from the radar is fixed at $r_t = 10$ m. N_s and BW are fixed at 300 K and 0.1 GHz respectively. Again, as before, $\sigma_{c_{avg}} = \sigma_{t_{avg}} = 1m^2$ while q is 2. As mentioned earlier, the first term in (7) shows the effect of SNR on the radar detection performance. Here, as the P_{tx} increases, the SNR improves resulting in higher detection performance. However, as pointed out in Corollary 1.3, when we enter the clutter limited scenario, increase in P_{tx} does not improve the performance of the radar since clutter returns proportionately increase. As a result, P_{DC} converges. The dotted line in the figure shows the P_{tx} value for which the P_{DC} converges. This was derived from the analytical expression in corollary 1.3 in (21). Note that the maximum power is independent of α and other clutter related terms.

Next, we run a similar study where we analyze the effect of the radar bandwidth on P_{DC} . The results are presented in Fig.4d for different α . In this scenario, P_{tx} is 30 dBm, N_s is 1500K while r_t is 10m and q is 2. All the other parameters are fixed as before except

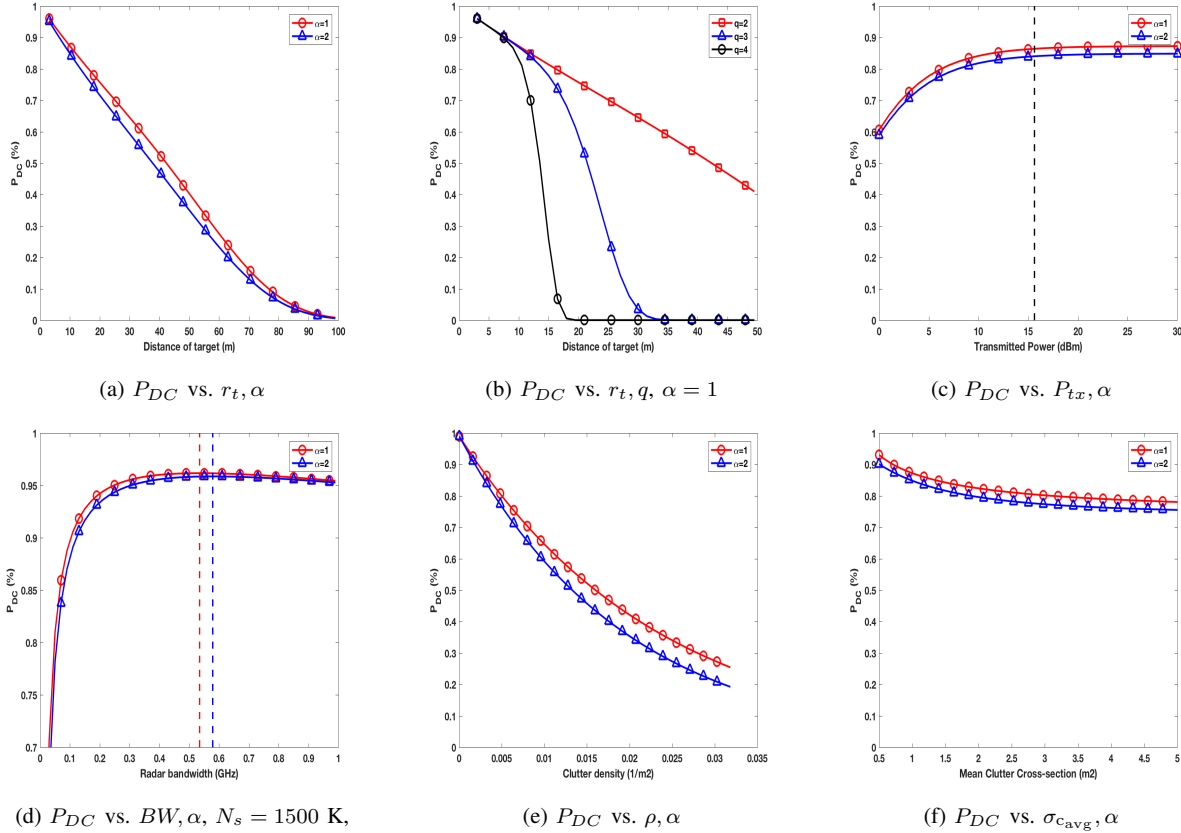


Fig. 4: Theorem 1 results: Variation of P_{DC} for different parameters when $r_f = 3$ m, $\sigma_{t_{avg}} = 1$ m², $\rho = 0.003$ /m². $P_{tx} = 30$ dBm for all figures except (c). $q = 2$ in all figures except (b). $r_t = 10$ m for figures (c)-(f). $BW = 0.1$ GHz, $N_s = 300$ K, in all figures except (d). $\sigma_{c_{avg}} = 1$ m² in all figures except (f).

for BW which is varied up to 1 GHz. In a clutter limited scenario, higher BW results in smaller range cells resulting in fewer clutter scatterers for a fixed clutter density. Hence, we have higher signal-to-clutter (SCR) levels and P_{DC} . On the other hand, in a noise limited scenario, increase in BW lowers the SNR and deteriorates the P_{DC} . Therefore, in the figure, we observe an optimum bandwidth where we obtain the peak P_{DC} . The two dotted lines indicate the optimum bandwidth derived from Theorem 2 (23), for $\alpha = 1$ (red left line) and $\alpha = 2$ (blue right line). This result shows the correspondence between theorems 1 and 2. The result shows P_{DC} improving with increase in BW (due to fall in noise) up to the optimum value after which the P_{DC} falls slightly for higher values of BW (due to fall in clutter returns).

Figure 4e shows the radar detection performance as a function of the clutter density (ρ) for a fixed target distance at $r_t = 10$ m. All the other radar parameters such as noise, bandwidth and transmitted power are fixed. We observe that, as expected, the increase in clutter density causes a fall in the SCR term in the second exponential term of (7). Finally, we consider the effect of $\sigma_{c_{avg}}$ on P_{DC} in Fig. 4f. Based on the figure we observe that the P_{DC} does fall up to a point with increase in $\sigma_{c_{avg}}$. However, it asymptotically converges after a point. In other words, the increase in $\sigma_{c_{avg}}$ does not affect P_{DC} beyond a point. This also concurs with the Corollaries 1.1 and 1.2.

B. Analysis of Results from Theorem 2

Now, we discuss the results obtained from the second theorem based on Fig. 5. A radar's SNR falls with increase in radar bandwidth due to noise. But the SCR improves due to smaller range cells

and fewer clutter scatterers. The optimum bandwidth is identified by Theorem 2 by considering both.

First, we show how the optimum radar bandwidth derived from (23) varies as a function of target distance r_t in Fig. 5a. Here, P_{tx} , N_s , ρ and q are fixed at 30 dBm, 300 K, 0.003 1/m² and 2 respectively. As we go farther, we observe that the optimum bandwidth reduces till it begins to converge. However, the variation in bandwidth is not considerable for high values of ranges. Therefore, it may be fairly easy to select an optimum bandwidth based on half the maximum unambiguous range of the radar field of view. The variation with respect to the clutter shape parameter is not very significant.

In Fig. 5b, we observe that the BW is far more sensitive to the clutter density ρ . Here, all the other parameters except ρ are fixed with the same values as the previous case. Higher clutter density requires a higher bandwidth or a smaller range cell size in order to balance the increase in noise. The returns are sensitive to the Weibull shape parameter. Slightly higher bandwidths are required for the Rayleigh distribution ($\alpha = 2$) compared to the exponential distribution ($\alpha = 1$).

In corollary 1.1, we observed that J becomes negligible for higher values of $\sigma_{c_{avg}}$. This is also observed in Fig. 5c. As a result, we observe how the optimum bandwidth for $\alpha = 1$ and $\alpha = 2$ begin to converge to the same value in Fig. 5d. This supports the corollary 2.1 where we noted that the optimum bandwidth becomes independent of the type of clutter distribution for high $\sigma_{c_{avg}}$.

As a result, we can tune the threshold function γ for a given transmitted power P_{tx} and radar bandwidth BW for obtaining the maximum P_{DC} while tracking a target at a specific target distance

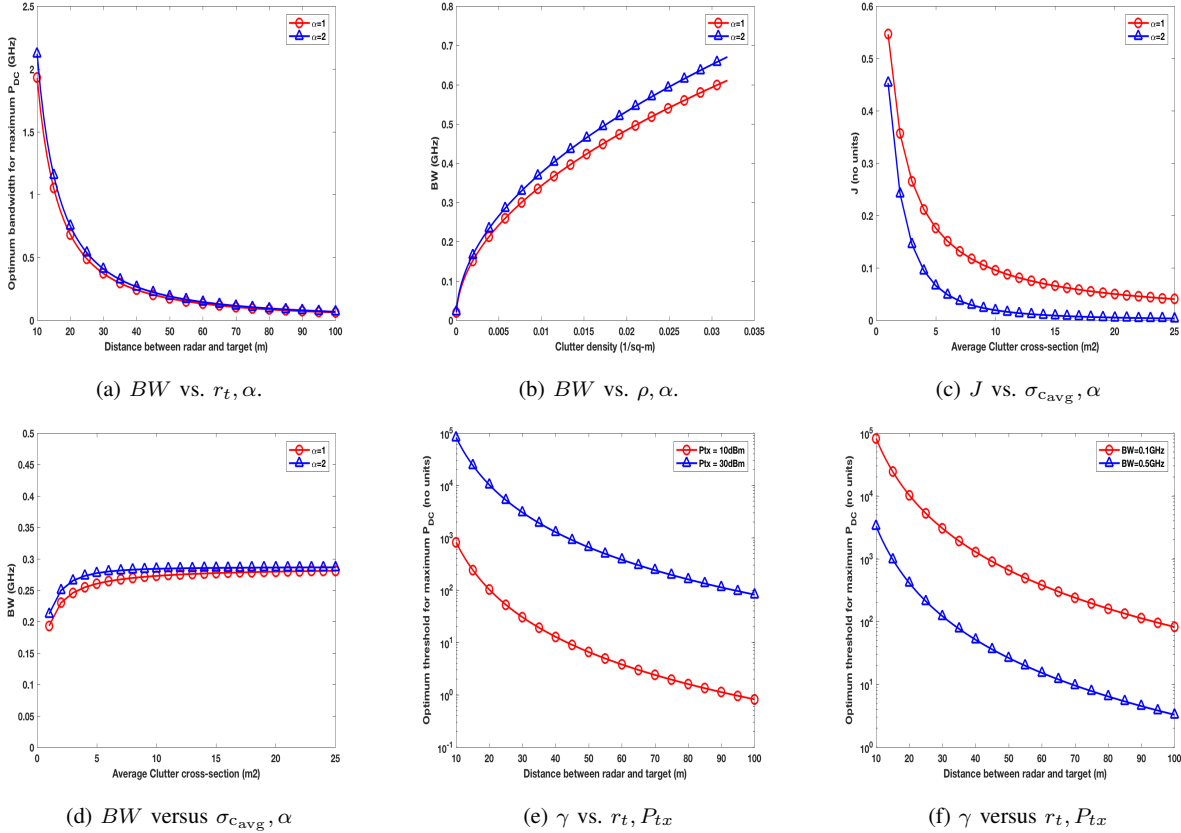


Fig. 5: Theorem 2 results: Optimum bandwidth for obtaining maximum P_{DC} when $N_s = 300$ K, $q = 2$, $\sigma_{t,avg} = 1$ m^2 . ρ fixed at 0.003 $1/m^2$ in all cases except (b). γ fixed at 1 in all cases except (e) and (f).

of r_t . Fig.5e shows the ideal γ as a function of r_t for different P_{tx} while keeping all other parameters fixed. We observe that γ has to be reduced as we go farther from the radar to compensate for the path loss factor. Greater transmitted powers result in higher γ as expected. Fig.5f shows how to tune γ for different bandwidths. Higher bandwidths require lower γ .

V. CONCLUSION

We derived a metric - radar detection coverage probability (P_{DC}) - using SG techniques for evaluating the radar's detection performance under generalized discrete clutter conditions. The point clutter distribution were modeled as a homogeneous Poisson point process. The RCS of the target and clutter scatterers were modelled as random variables of Swerling-1 and Weibull distributions respectively. We evaluated the radar's performance for different radar, target and clutter parameters. Our studies provide several insights into the performance of the radar under noisy and cluttered conditions. We list some of these below:

- The radar detection performance is best for the exponential distribution of the clutter cross-section (when Weibull shape parameter $\alpha = 1$) and worst for the Rayleigh distribution ($\alpha = 1$) for similar clutter densities and mean clutter RCS.
- Increase in radar transmitted power improves the detection performance in noise limited scenarios. However, beyond a point there is no further discernible improvement since we enter the clutter limited scenario. We provide the analytical method to estimate the peak transmitted power at which the radar detection performance reaches its asymptotic maximum.

- Large radar bandwidths result in increase in noise but lesser clutter returns due to smaller range cell size. We derive the optimum radar bandwidth for maximizing P_{DC} under noisy and cluttered conditions.
- We show a method for optimizing the detection threshold for maximizing P_{DC} for a fixed transmitted power and bandwidth.

Our results are experimentally validated with a hybrid of FDTD and Monte Carlo simulations. The FDTD simulations are used to model the path loss between the radar and a scatterer while the Monte Carlo simulations consider the diversity in the RCS of the target and clutter scatterers.

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